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ON THE NUMBER OF N–DIMENSIONAL INVARIANT SPHERES IN POLYNOMIAL VECTOR FIELDS OF $\mathbb{C}^{N+1}$

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Abstract We study the polynomial vector fields $\mathcal{X} = \sum_{i=1}^{n+1} P_i(x_1, \ldots, x_{n+1}) \frac{\partial}{\partial x_i}$ in $\mathbb{C}^{n+1}$ with $n \geq 1$. Let m_i be the degree of the polynomial P_i. We call $(m_1, \ldots, m_{n+1})$ the degree of $\mathcal{X}$. For these polynomial vector fields $\mathcal{X}$ and in function of their degree we provide upper bounds, first for the maximal number of invariant n–dimensional spheres, and second for the maximal number of n–dimensional concentric invariant spheres.

Keywords polynomial vector fields, invariant spheres, invariant circles, extactic algebraic hypersurface.

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1. Introduction and statement of the main results

Let $\mathcal{X}$ be the *polynomial vector field* in $\mathbb{C}^{n+1}$ defined by

$$\mathcal{X} = \sum_{i=1}^{n+1} P_i(x_1, \ldots, x_{n+1}) \frac{\partial}{\partial x_i},$$

where every P_i is a polynomial of degree m_i in the variables $x_1, \ldots, x_{n+1}$ with coefficients in $\mathbb{C}$. We say that $\mathbf{m} = (m_1, \ldots, m_{n+1})$ is the *degree* of the polynomial field, we assume without loss of generality that $m_1 \geq \cdots \geq m_{n+1}$. We recall that the *polynomial differential system* in $\mathbb{C}^{n+1}$ of degree $\mathbf{m}$ associated with the vector field $\mathcal{X}$ is written as

$$\frac{dx_i}{dt} = P_i(x_1, \ldots, x_{n+1}), \qquad i = 1, \ldots, n+1.$$

By the Darboux theory of integrability we know that the existence of a sufficiently large number of invariant algebraic hypersurfaces guarantees the existence of a first integral for the polynomial vector field $\mathcal{X}$ which can be calculated explicitly, see for instance [4, 6]. As usual $\mathbb{C}[x_1, \ldots, x_{n+1}]$ denotes the ring of all polynomials in the variables $x_1, \ldots, x_{n+1}$ and coefficients in $\mathbb{C}$. We recall that an *invariant algebraic*

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