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On the number of invariant conics for the polynomial vector fields defined on quadrics

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Abstract

The quadrics here considered are the nine real quadrics: parabolic cylinder, elliptic cylinder, hyperbolic cylinder, cone, hyperboloid of one sheet, hyperbolic paraboloid, elliptic paraboloid, ellipsoid and hyperboloid of two sheets. Let $\mathcal{Q}$ be one of these quadrics. We consider a polynomial vector field $\mathcal{X} = (P, Q, R)$ in $\mathbb{R}^3$ whose flow leaves $\mathcal{Q}$ invariant. If $m_1 =$ degree P, $m_2 =$ degree Q and $m_3 =$ degree R, we say that $\mathbf{m} = (m_1, m_2, m_3)$ is the degree of $\mathcal{X}$. In function of these degrees we find a bound for the maximum number of invariant conics of $\mathcal{X}$ that results from the intersection of invariant planes of $\mathcal{X}$ with $\mathcal{Q}$. The conics obtained can be degenerate or not. Since the first six quadrics mentioned are ruled surfaces, the degenerate conics obtained are formed by a point, a double straight line, two parallel straight lines, or two intersecting straight lines; thus for the vector fields defined on these quadrics we get a bound for the maximum number of invariant straight lines contained in invariant planes of $\mathcal{X}$. In the same way, if the conic is non-degenerate, it can be a parabola, an ellipse or a hyperbola and we provide a bound for the maximum number of invariant non-degenerate conics of the vector field $\mathcal{X}$ depending on each quadric $\mathcal{Q}$ and of the degrees m_1, m_2 and m_3 of $\mathcal{X}$.

MSC: 34A34; 34C05; 34C14

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