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Periodic Solutions of a Class of Second-Order Differential Equation

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Abstract

We study the periodic solutions of the second-order differential equations of the form

$$\ddot{x}+3x\dot{x}+x^3+F(t)\left(\dot{x}+x^2\right)+G(t)x+H(t)=0,$$

where the functions $F(t)$, $G(t)$ and $H(t)$ are periodic of period 2π in the variable *t*.

Keywords

Periodic Solution, Differential Equation, Averaging Theory

1. Introduction and Statement of the Main Results

In this paper we shall study the existence of periodic solutions of the second-order differential equation of the form

$$\ddot{x}+3x\dot{x}+x^3+F(t)\left(\dot{x}+x^2\right)+G(t)x+H(t)=0, \tag{1}$$

where the dot denotes derivative with respect to the time *t*, and the functions $F(t)$, $G(t)$ and $H(t)$ are periodic of period 2π in the variable *t*.

We note that the second-order differential Equation (1), when $F=G=H=0$, appears in the Ince's catalog of equations possessing the Painlevé property (see [1]). Moreover, the differential equation $\ddot{x}+3x\dot{x}+x^3=0$ is well known in many areas of mathematics and physics, and it possesses the algebra $\mathrm{sl}(3,\mathbb{R})$ of Lie point symmetries (see for more details in the paper [2] and the references quoted there).

In a recent paper [3] (see also [4] [5]), the second-order differential Equation (1) has been studied when $F=H=0$. A study of coupled quadratic unharmonic oscillators in terms of the Painlevé analysis and inte-

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