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Homeomorphisms Between Limbs of the Mandelbrot Set

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ABSTRACT. Using a family of higher degree polynomials as a bridge, together with complex surgery techniques, we construct a homeomorphism between any two limbs of the Mandelbrot set of equal denominator. Induced by these homeomorphisms and complex conjugation, we obtain an involution between each limb and itself, whose fixed points form a topological arc. All these maps have counterparts at the combinatorial level relating corresponding external arguments. Assuming local connectivity of the Mandelbrot set we may conclude that the constructed homeomorphisms between limbs are compatible with the embeddings of the limbs in the plane. As usual we plough in the dynamical planes and harvest in the parameter space.

1. Introduction

The Mandelbrot set has been a main object of interest and study in recent years. It is associated with the quadratic family of complex polynomials $Q_c(z) = z^2 + c$, for $c \in \mathbb{C}$. Let K_c denote the *filled Julia set* of Q_c, i.e.,

$$K = K_c = \left\{ z \in \mathbb{C} \mid \left\{ Q_c^n(z) \right\}_{n \in \mathbb{N}} \text{ is bounded } \right\} ,$$

and let $J = J_c$ be the boundary of K_c, the *Julia set* of Q_c.

As for all rational functions, the dynamical behavior of the critical point $\omega = 0$ dominates the dynamical behavior of the polynomial. The filled Julia set K_c is connected when the orbit of 0 is bounded, and it is totally disconnected when it is unbounded.

The Mandelbrot set, M, is defined as the set of parameter values c, for which K_c is connected, or equivalently as the set of parameters for which the orbit of 0 is bounded.

The works of Douady and Hubbard [10, 11], Mañe, et al. [20], and Yoccoz in the last decade contributed enormously to the understanding of the Mandelbrot set, but there remain many interesting open questions. The main one is to prove that M is locally connected (MLC conjecture). The following are some well-known results about M:

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