

EXTENSIONS OF HOMEOMORPHISMS BETWEEN LIMBS OF THE MANDELBROT SET

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ABSTRACT. Using holomorphic surgery techniques, we construct a homeomorphism between a neighborhood of any limb without root point of the Mandelbrot set and a neighborhood of any other of equal denominator, in such a way that the limbs are mapped to each other. On the limbs, the homeomorphism coincides with that constructed in “Homeomorphisms between limbs of the Mandelbrot set” (*J. Geom. Anal.* **9** (1999), 327–390) which proves – without assuming local connectivity of the Mandelbrot set – that these maps are compatible with the embedding of the limbs in the plane. Outside the limbs, the constructed extension is quasi-conformal.

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