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LIMIT CYCLES FOR SOME ABEL EQUATIONS HAVING COEFFICIENTS WITHOUT FIXED SIGNS

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We prove that some 2π-periodic generalized Abel equations of the form $x' = A(t)x^n + B(t)x^m + C(t)x$, with $n \neq m$ and $n, m \geq 2$ have at most three limit cycles. The novelty of our result is that, in contrast with other results of the literature, our hypotheses allow the functions A, B, and C to change sign. Finally we study in more detail the Abel equation $x' = A(t)x^3 + B(t)x^2$, where the functions A and B are trigonometric polynomials of degree one.

Keywords: Abel equation; periodic solution; limit cycle.

1. Main Results

In this work we consider nonautonomous differential equations of the form

$$\frac{dx}{dt} = A_0(t) + A_1(t)x + \cdots + A_{k-1}(t)x^{k-1} + A_k(t)x^k, \tag{1}$$

where the functions A_j, $j = 0, 1, \ldots, k$ are real valued, 2π-periodic, and continuous and $x, t \in \mathbb{R}$. In particular when $k = 3$, they are the well-known Abel equations. The solutions of (1) satisfying $x(0) = x(2\pi)$ are usually called *periodic solutions*. A periodic solution which is isolated from other periodic solutions of (1) is called a *limit cycle* of the differential equation. It is so labeled since these solutions are actual limit cycles of (1) when it is considered on a cylinder. The interest in the study of these solutions is due to its relation with the Hilbert sixteenth problem about the number of limit cycles of planar polynomial autonomous equations, see, for instance, [Carbonell & Llibre, 1988; Cherkas, 1976; Devlin *et al.*, 1998; Gasull & Guillamon, 2006; Gasull & Llibre, 1990; Gasull *et al.*, 2005; Lins-Neto, 1980].

One of the most important papers in this subject is that of Lins-Neto [1980], where the author proves that, given $\ell \in \mathbb{N}$, there exist $A_2(t)$, $A_3(t)$, trigonometric polynomials such that

$$x' = A_2(t)x^2 + A_3(t)x^3, \tag{2}$$

has at least ℓ limit cycles and, on the other hand, when $A_3(t)$ does not change sign, then it has at most three limit cycles.

Therefore, a natural problem is to obtain conditions on functions $A_j(t)$ in (1) that allow to give explicit upper bounds for the number of