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A sufficient condition in order that the real Jacobian conjecture in $\mathbb{R}^2$ holds

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Abstract

Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that $\det DF(x, y)$ is different from zero for all $(x, y) \in \mathbb{R}^2$ and $F(0, 0) = (0, 0)$. We prove that for the injectivity of F it is sufficient to assume that the higher homogeneous terms of the polynomials $ff_x + gg_x$ and $ff_y + gg_y$ do not have real linear factors in common. The proofs are based on qualitative theory of dynamical systems.

MSC: primary 14R15; secondary 35F05, 35A30

Keywords: Real Jacobian conjecture; Global injectivity; Centre

1. Introduction and statement of the main result

Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a smooth map such that $\det DF(x, y)$ is different from zero for all $(x, y) \in \mathbb{R}^2$. It is clear that F is a local diffeomorphism, but it is not always injective. There are very general well known additional conditions to ensure that F is a global diffeomorphism, see for instance [8,11,13].

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