ISOCHRONICITY FOR TRIVIAL QUINTIC AND SEPTIC PLANAR POLYNOMIAL HAMILTONIAN SYSTEMS

FRANCISCO BRAUN, JAUME LLIBRE AND ANA C. MEREU

ABSTRACT. In this paper we completely characterize trivial isochronous centers of degrees 5 and 7. Precisely, we provide formulas, up to linear change of coordinates, for the Hamiltonian H of the isochronous centers such that $H = \left(f_1^2 + f_2^2\right)/2$ has degrees 6 and 8, and $f = (f_1, f_2) : \mathbb{R}^2 \to \mathbb{R}^2$ is a polynomial map with $\det Df = 1$ and $f(0,0) = (0,0)$.

1. INTRODUCTION

Let $P(x, y)$ and $Q(x, y)$ be real polynomials in the variables x and y. We say that a polynomial vector field $\mathcal{X} = (P, Q)$ has degree n when $\max\{\deg P, \deg Q\} = n$. Given a polynomial Hamiltonian $H : \mathbb{R}^2 \to \mathbb{R}$ of degree $n+1$, the associated polynomial Hamiltonian system of degree n is

$$\dot{x} = -H_y(x, y), \quad \dot{y} = H_x(x, y). \tag{1}$$

System (1) has a *center* at $(0,0)$ if there is a neighbourhood of the origin filled of periodic orbits except the origin. The maximum connected set filled of periodic orbits having in its inner boundary the origin is called the *period annulus* of the center localized at the origin. If the period annulus is $\mathbb{R}^2 \setminus \{(0,0)\}$, we call the center *global*. We say that a polynomial Hamiltonian system has an *isochronous center* at the origin if $(0,0)$ is a center of (1) and all the orbits in the period annulus of the center have the same period.

The following characterization of the polynomial Hamiltonian systems possessing an isochronous center at the origin was given in [6]. The polynomial Hamiltonian system (1) has an isochronous center of period 2π at the origin if and only if

$$H(x, y) = \frac{f_1(x, y)^2 + f_2(x, y)^2}{2}, \tag{2}$$

for all (x, y) in a neighborhood N_0 of the origin, where $f = (f_1, f_2) : N_0 \to \mathbb{R}^2$ is an analytic map with Jacobian determinant $\det Df$ constant and equal to 1, and $f(0,0) = (0,0)$. We observe that this characterization still holds for analytic Hamiltonians. When f can be taken polynomial, we say that

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