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Abel-like differential equations with no periodic solutions ☆

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Abstract

We present various criteria for the non-existence of positive periodic solutions of generalized Abel differential equations with periodic coefficients that can change sign. As an application, we obtain some families of planar vector fields without limit cycles.

Keywords: Abel equation; Limit cycles; Hilbert's 16th problem; Rigid systems; Periodic solutions

1. Introduction

Hilbert's 16th problem [12] is usually stated as determining the maximum number of limit cycles (isolated periodic orbits) in terms of the degrees of a polynomial system in the plane

$$\begin{cases} x' = P(x, y), \\ y' = Q(x, y), \end{cases} \tag{1.1}$$

where P and Q are polynomials. Although there has long been intense research interest in this problem, only recently has it been proved that the number of limit cycles is finite for each individual equation [6,13].

Bounds on the number of limit cycles have only been found for some families of polynomial systems, the problems most extensively studied being non-existence and uniqueness. In most cases, a change of variables proposed by Cherkas [4] is used to obtain an equivalence between the number of limit cycles of (1.1) for some P and Q, and the number of positive periodic solutions of an Abel-like differential equation

$$x' = \sum_{i=0}^{n} A_i(t) x^i, \tag{1.2}$$

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