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# PERIODIC SOLUTIONS OF NONLINEAR PERIODIC DIFFERENTIAL SYSTEMS WITH A SMALL PARAMETER

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Abstract. We deal with nonlinear periodic differential systems depending on a small parameter. The unperturbed system has an invariant manifold of periodic solutions. We provide sufficient conditions in order that some of the periodic orbits of this invariant manifold persist after the perturbation. These conditions are not difficult to check, as we show in some applications. The key tool for proving the main result is the Lyapunov–Schmidt reduction method applied to the Poincaré–Andronov mapping.

1. **Introduction.** We consider the problem of bifurcation of $T$–periodic solutions for a differential system of the form,

$$x'(t) = F_0(t,x) + \varepsilon F_1(t,x) + \varepsilon^2 R(t,x,\varepsilon), \tag{1}$$

where $\varepsilon$ is a small parameter, $F_0, F_1 : \mathbb{R} \times \Omega \to \mathbb{R}^n$ and $R : \mathbb{R} \times \Omega \times (-\varepsilon_f, \varepsilon_f) \to \mathbb{R}^n$ are $C^2$ functions, $T$–periodic in the first variable, and $\Omega$ is an open subset of $\mathbb{R}^n$. One of the main hypotheses is that the unperturbed system

$$x'(t) = F_0(t,x), \tag{2}$$

has a manifold of periodic solutions. This problem was solved before by Malkin (1956) and Roseau (1966) (see [4]). We will give here a new and shorter proof (see Theorem 3.1 and its proof). In addition, we will give a series of corollaries in some particular cases. In order to describe these cases we introduce some notation. We denote the projection onto the first $k$ coordinates by $\pi : \mathbb{R}^k \times \mathbb{R}^{n-k} \to \mathbb{R}^k$ and the one onto the last $(n-k)$ coordinates by $\pi^\perp : \mathbb{R}^k \times \mathbb{R}^{n-k} \to \mathbb{R}^{n-k}$. For the

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