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J. Math. Anal. Appl. 331 (2007) 443-454

Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS

www.elsevier.com/locate/jmaa

## The third order Melnikov function of a quadratic center under quadratic perturbations $\stackrel{\text{\tiny{$x$}}}{=}$

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Received 25 May 2006

Available online 26 September 2006

Submitted by Steven G. Krantz

## Abstract

We study quadratic perturbations of the integrable system (1 + x) dH, where  $H = (x^2 + y^2)/2$ . We prove that the first three Melnikov functions associated to the perturbed system give rise at most to three limit cycles.

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Keywords: Quadratic systems; Limit cycles; Bifurcation; High order Melnikov functions

## 1. Introduction and statement of the main result

Planar vector fields  $\dot{x} = X(x, y)$ ,  $\dot{y} = Y(x, y)$  defined in the real plane, when X(x, y) = 0and Y(x, y) = 0 are arbitrary conics, are usually called quadratic systems. The Hilbert sixteenth problem [5] restricted to them asks for the number and distribution of limit cycles inside this family. It is known that each limit cycle must surround a unique singularity of focus type, that at

<sup>\*</sup> The first author is partially supported by Agence universitaire de la Francophonie; the second author is partially supported by a MCYT grant MTM2005-06098-C02-01 and by a CIRIT grant number 2005SGR 00550; the third author is partially supported by the NSFC project number 10571002 of China. This paper is also supported by the CRM Research Programme: On Hilbert's 16th Problem.

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