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A second order analysis of the periodic solutions for nonlinear periodic differential systems with a small parameter

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ABSTRACT

We deal with nonlinear T-periodic differential systems depending on a small parameter. The unperturbed system has an invariant manifold of periodic solutions. We provide the expressions of the bifurcation functions up to second order in the small parameter in order that their simple zeros are initial values of the periodic solutions that persist after the perturbation. In the end two applications are done. The key tool for proving the main result is the Lyapunov–Schmidt reduction method applied to the T-Poincaré–Andronov mapping.

1. Introduction

We want to study the existence of T-periodic solutions of the differential systems of the form,

$$x'(t) = F_0(t,x) + \varepsilon F_1(t,x) + \varepsilon^2 F_2(t,x) + \varepsilon^3 R(t,x,\varepsilon), \tag{1}$$

where ε is a small parameter, $F_0, F_1, F_2 : \mathbb{R} \times \Omega \to \mathbb{R}^n$ and $R : \mathbb{R} \times \Omega \times (-\varepsilon_f, \varepsilon_f) \to \mathbb{R}^n$ are C^2 functions, T-periodic in the first variable, and Ω is an open subset of $\mathbb{R}^n$. We work in the hypothesis that there exists a k-dimensional submanifold of Ω $(k \le n)$ whose points are initial values of T-periodic solutions of the unperturbed system

$$x'(t) = F_0(t,x). \tag{2}$$

Our objective is to study the periodic solutions of the unperturbed system (2) which can be continued to the perturbed system (1) for values of ε sufficiently small.

For $z \in \Omega$ we denote by $x(\cdot, z, \varepsilon) : [0, t_{(z,\varepsilon)}) \to \mathbb{R}^n$ the solution of (1) with $x(0,z,\varepsilon) = z$. From Theorem 8.3 of [1] we deduce that, whenever $t_{(z_0,0)} > T$ for some $z_0 \in \Omega$ there exists a neighborhood of $(z_0, 0)$ in $\Omega \times (-\varepsilon_f, \varepsilon_f)$ such that, for all (z,ε) in this neighborhood, $t_{(z,\varepsilon)} > T$. Under this assumption there exists an open subset D of Ω and a sufficiently small $\varepsilon_0 > 0$ such that, for all $(z,\varepsilon) \in D \times (-\varepsilon_0, \varepsilon_0)$, the solution $x(\cdot, z, \varepsilon)$ is defined on the interval $[0, T]$. Hence, we can consider the function $f : D \times (-\varepsilon_0, \varepsilon_0) \to \mathbb{R}^n$, given by

$$f(z,\varepsilon) = x(T, z, \varepsilon) - z. \tag{3}$$

Then, every $(z_\varepsilon, \varepsilon)$ such that

$$f(z_\varepsilon, \varepsilon) = 0 \tag{4}$$

provides the periodic solution $x(\cdot, z_\varepsilon, \varepsilon)$ of (1).

The converse is also true, i.e. for every T-periodic solution of (1), if we denote by z_ε its value at $t = 0$ then (4) holds. Then, the problem of finding a T-periodic solution of (1), can be replaced by the problem of finding zeros of the finite-dimensional function $f(\cdot, \varepsilon)$ given by (3).

We denote the variational equation of (2) associated to one of its solutions $x(t, z, 0)$ with

$$y' = P(t,z)y, \tag{5}$$

where

$$P(t,z) = D_x F_0(t, x(t,z,0)), \tag{6}$$

and with $Y(\cdot, z)$ some fundamental matrix solution of (5).

We denote the projection onto the first k coordinates by $\pi : \mathbb{R}^k \times \mathbb{R}^{n-k} \to \mathbb{R}^k$ and the one onto the last $(n-k)$ coordinates by $\pi^\perp : \mathbb{R}^k \times \mathbb{R}^{n-k} \to \mathbb{R}^{n-k}$. For the n-dimensional function g of n variables $z = (\alpha, \beta) \in \mathbb{R}^k \times \mathbb{R}^{n-k}$, we denote by $D_\beta(\pi g)$ the $k \times (n-k)$ matrix whose entries are the first order partial derivatives with respect to each component of $\beta \in \mathbb{R}^{n-k}$ of the first k components of g.

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