Topological Methods in Nonlinear Analysis Volume 43, No. 2, 2014, 403–419

C2014 Juliusz Schauder Centre for Nonlinear Studies Nicolaus Copernicus University

PERIODIC SOLUTIONS FOR NONLINEAR DIFFERENTIAL SYSTEMS: THE SECOND ORDER BIFURCATION FUNCTION

Adriana Buică — Jaume Giné — Jaume Llibre

ABSTRACT. We are concerned here with the classical problem of Poincaré of persistence of periodic solutions under small perturbations. The main contribution of this work is to give the expression of the second order bifurcation function in more general hypotheses than the ones already existing in the literature. We illustrate our main result constructing a second order bifurcation function for the perturbed symmetric Euler top.

1. Introduction

We are concerned here with the classical problem of Poincaré of persistence of periodic solutions under small perturbations. More precisely, we consider a family of T-periodic, sufficiently smooth, n-dimensional systems of the form

(1.1)
$$x'(t) = F(t, x, \varepsilon),$$

depending on a small (perturbation) parameter ε . We assume that there exists some nonempty set \mathcal{Z} whose points are initial values for *T*-periodic solutions of the unperturbed system

(1.2)
$$x'(t) = F(t, x, 0).$$

Key words and phrases. Periodic solution, Lyapunov–Schmidt reduction, period manifold, small parameter, the second order bifurcation function.

²⁰¹⁰ Mathematics Subject Classification. Primary 34C29, 34C25; Secondary 58F22.