



ELSEVIER

Bull. Sci. math. 128 (2004) 7–22

**BULLETIN  
DES SCIENCES  
MATHÉMATIQUES**

www.elsevier.com/locate/bulsci

# Averaging methods for finding periodic orbits via Brouwer degree

Adriana Buică<sup>a,1</sup>, Jaume Llibre<sup>b,\*</sup>

<sup>a</sup> *Department of Applied Mathematics, Babeş-Bolyai University of Cluj-Napoca, Romania*

<sup>b</sup> *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*

Received 15 May 2003; accepted 4 September 2003

---

## Abstract

We consider the problem of finding  $T$ -periodic solutions for a differential system whose vector field depend on a small parameter  $\varepsilon$ . An answer to this problem can be given using the averaging method. Our main results are in this direction, but our approach is new. We use topological methods based on Brouwer degree theory to solve operator equations equivalent to this problem. The regularity assumptions are weaker then in the known results (up to second order in  $\varepsilon$ ). A result for third order averaging method is also given.

As an application we provide a way to study bifurcations of limit cycles from the period annulus of a planar system and notice relations with the displacement function. A concrete example is given. © 2003 Elsevier SAS. All rights reserved.

*MSC:* 34C29; 34C25; 47H11

*Keywords:* Periodic solution; Averaging method; Brouwer degree

---

## 1. Introduction

In few words we can say that the averaging method [15,16] gives a quantitative relation between the solutions of some non autonomous differential system and the solutions of the averaged differential system, which is an autonomous one. The averaging is with respect to the independent variable and the right hand sides of these systems are sufficiently small, depending on a small parameter  $\varepsilon$ . Also, by using the Implicit Function Theorem, the

---

\* Corresponding author.

*E-mail addresses:* fbuica@crm.es (A. Buică), jllibre@mat.uab.es (J. Llibre).

<sup>1</sup> Current address: Centre de Recerca Matemàtica, 08193 Bellaterra, Barcelona, Spain.