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BIFURCATION OF LIMIT CYCLES FROM A FOUR-DIMENSIONAL CENTER IN CONTROL SYSTEMS

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We study the bifurcation of limit cycles from the periodic orbits of a four-dimensional center in a class of piecewise linear differential systems, which appears in a natural way in control theory. Our main result shows that three is an upper bound for the number of limit cycles, up to first-order expansion of the displacement function with respect to the small parameter. Moreover, this upper bound is reached. For proving this result we use the averaging method in a form where the differentiability of the system is not needed.

Keywords: Limit cycles; piecewise linear differential systems; four-dimensional control system.

1. Introduction

Piecewise linear differential systems appear in a natural way in control theory. These systems can present the complicated dynamical phenomena as the general nonlinear differential systems. Some of the main ingredients in the qualitative description of the dynamical behavior of differential systems are the number and distribution of the limit cycles.

The purpose of this paper is to study the existence of limit cycles of the four-dimensional control system

$$x' = A_0 x + \varepsilon F(x), \tag{1}$$

for $|\varepsilon| \neq 0$ a sufficiently small real parameter, where

$$A_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

and $F : \mathbb{R}^4 \to \mathbb{R}^4$ is given by

$$F(x) = Ax + \varphi(k^T x)b,$$

with $A \in \mathcal{M}_4(\mathbb{R})$, $k, b \in \mathbb{R}^4 \backslash \{0\}$ and $\varphi : \mathbb{R} \to \mathbb{R}$ the piecewise linear function

$$\varphi(x) = \begin{cases} -1, & \text{for } x \in (-\infty, -1), \\ x, & \text{for } x \in [-1, 1], \\ 1, & \text{for } x \in (1, \infty). \end{cases} \tag{2}$$

The independent variable is denoted by t, vectors of $\mathbb{R}^4$ are column vectors, and k^T denotes the transposed vector.

For $\varepsilon = 0$ system (1) becomes

$$x_1' = -x_2, \quad x_2' = x_1, \quad x_3' = -x_4, \quad x_4' = x_3. \tag{3}$$

We notice that the origin is a *global isochronous center* for (3), i.e. all its orbits are periodic with the same period. A *limit cycle* is an isolated periodic orbit in a set of periodic orbits. The *Poincaré*

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