SECOND BOGOLUBOV'S THEOREM FOR LIPSCHITZ SYSTEMS AND BIFURCATIONS OF ASYMPTOTICALLY STABLE PERIODIC SOLUTIONS IN DIFFERENTIAL EQUATIONS WITH JUMPING NONLINEARITIES

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The goal of this paper is to study bifurcations of asymptotically stable 2π-periodic solutions in the forced asymmetric oscillator $\ddot{u} + \varepsilon c\dot{u} + u + \varepsilon a u^+ = \varepsilon\lambda\cos t$ by means of a Lipschitz generalization of the second Bogolubov's theorem due to the authors. The small parameter $\varepsilon > 0$ is introduced in such a way that any solution of the system corresponding to $\varepsilon = 0$ is 2π-periodic. We show that exactly one of these solutions whose amplitude is $2|\lambda|/\sqrt{a^2+4c^2}$ generates a branch of 2π-periodic solutions when $\varepsilon > 0$ increases. The solutions of this branch are asymptotically stable provided that $c > 0$.

Keywords: Differential equations with jumping nonlinearities; Asymptotically stable periodic solutions; Bifurcation.

1. Introduction

The differential equation for the coordinate u of the mass attached via nonlinear spring to an immovable beam drawn at Fig. 1 is written down as