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Bifurcations from nondegenerate families of periodic solutions in Lipschitz systems

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ABSTRACT

The paper addresses the problem of bifurcation of periodic solutions from a normally nondegenerate family of periodic solutions of ordinary differential equations under perturbations. The approach to solve this problem can be described as transforming (by a Lyapunov–Schmidt reduction) the initial system into one which is in the standard form of averaging, and subsequently applying the averaging principle. This approach encounters a fundamental problem when the perturbation is only Lipschitz (nonsmooth) as we do not longer have smooth Lyapunov–Schmidt projectors. The situation of Lipschitz perturbations has been addressed in the literature lately and the results obtained conclude the existence of the bifurcated branch of periodic solutions. Motivated by recent challenges in control theory, we are interested in the uniqueness problem. We achieve this in the case when the Lipschitz constant of the perturbation obeys a suitable estimate.

1. Introduction

In [21] Malkin developed a perturbation theory to study the existence, uniqueness and stability of T-periodic solutions in the n-dimensional T-periodic systems of the form

$$\dot{x} = f(t,x) + \varepsilon g(t,x,\varepsilon), \tag{1}$$

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