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ORIGINAL RESEARCH

A Note on Forced Oscillations in Differential Equations with Jumping Nonlinearities

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Abstract The goal of this paper is to study bifurcations of asymptotically stable 2π-periodic solutions in the forced asymmetric oscillator $\ddot{u}+\varepsilon c\dot{u}+u+\varepsilon a u^{+}=1+\varepsilon\lambda\cos t$ by means of a Lipschitz generalization of the second Bogolubov's theorem due to the authors. The small parameter $\varepsilon>0$ is introduced in such a way that any solution of the system corresponding to $\varepsilon=0$ is 2π-periodic. We show that exactly one of these solutions whose amplitude is $\frac{\lambda}{\sqrt{a^2+c^2}}$ generates a branch of 2π-periodic solutions when $\varepsilon>0$ increases. The solutions of this branch are asymptotically stable provided that $c>0$.

Keywords Asymptotic stability · Periodic solutions · Jumping nonlinearity · Method of averaging

Introduction

The differential equation for the coordinate u of the mass attached via nonlinear spring to an immovable beam drawn at Fig. 1 reads as

$$m\ddot{u}+c\dot{u}+k_1u+k_2u^{+}=f(t), \tag{1}$$

where f is a force applied to the mass in the vertical direction, see [1,11,15].

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