



New families of periodic orbits for a galactic potential



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ABSTRACT

We find analytically new families of periodic orbits of a Hamiltonian system which describes the local motion in the central area of a galaxy, whose dynamics have been studied by many authors.

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1. Introduction and statement of the main results

In this paper we study the families of periodic orbits of a 3-dimensional (or simply 3D) isotropic harmonic oscillator perturbed by a polynomial potential

$$\mathcal{H} = \frac{1}{2}(p_1^2 + p_2^2 + p_3^2) + \frac{1}{2}(Q_1^2 + Q_2^2 + Q_3^2) + \varepsilon \mathcal{P}(Q_1, Q_2, Q_3), \quad (1)$$

where ε is a small parameter. The polynomial potential is

$$\mathcal{P}(Q_1, Q_2, Q_3) = Q_1^4 + Q_2^4 + Q_3^4 + a(Q_1^2 Q_2^2 + Q_1^2 Q_3^2 + Q_2^2 Q_3^2), \quad (2)$$

where $a \in \mathbb{R}$ is a parameter.

The potential here studied

$$V = \frac{1}{2}(Q_1^2 + Q_2^2 + Q_3^2) + \varepsilon(Q_1^4 + Q_2^4 + Q_3^4 + a(Q_1^2 Q_2^2 + Q_1^2 Q_3^2 + Q_2^2 Q_3^2))$$

is a 3-dimensional perturbed harmonic oscillator and describes the local motion in the central area of a galaxy. These local 2-or 3-dimensional potentials, become of the expansion of global galactic potentials in a Taylor series near a stable equilibrium point and have been extensively studied in order to describe the local motion inside the galaxies. This potential has been studied by many authors, see for instance Deprit and Elipe [4], Caranicolas [3], Elipe and Deprit [5], Elipe [6], Arribas et al. [2], Zotos [11–14], Zotos and Caranicolas [15], Zotos and Carpintero [16] and others.

In paper [10] the authors studied analytically the families of periodic orbits of the Hamiltonian (1) with (2) using the averaging theory, and they find several families of periodic orbits. Here we improve the results of [10] finding new families of periodic orbits, also using a result based in the averaging theory. The key point for obtaining these new families of

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