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Discussion

Discussion on the limit cycles of the Lev Ginzburg equation by M. Bellamy and R.E. Mickens, Journal of Sound and Vibration 308 (2007) 337–342

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ABSTRACT

The authors M. Bellamy and R.E. Mickens in the article "Hopf bifurcation analysis of the Lev Ginzburg equation" published in Journal of Sound and Vibration 308 (2007) 337–342, claimed that this differential equation in the plane can exhibit a limit cycle. Here we prove that the Lev Ginzburg differential equation has no limit cycles.

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1. Introduction

In his studies on population dynamics in 1986 Ginzburg [1] worked with the following family of second order differential equations:

$$\frac{d^2x}{dt^2}+\alpha\left(1-\beta_1\frac{dx}{dt}\right)x=\left(1-\beta_1\frac{dx}{dt}\right)\left(\gamma+\beta\frac{dx}{dt}\right), \tag{1}$$

depending on four parameters: $\alpha>0$, $\beta_1>0$, $\gamma>0$ and $\beta\in\mathbb{R}$. Recently Bellamy and Mickens [2] claimed that the Lev Ginzburg differential equation (1) can exhibit a limit cycle coming from a Hopf bifurcation. We will show that this differential equation has neither a Hopf bifurcation, nor limit cycles.

The second order differential equation (1) can be written as the following first order planar polynomial differential system:

$$x'=\frac{dx}{dt}=y,$$

$$y'=\frac{dy}{dt}=(1-\beta_1 y)(\gamma-\alpha x+\beta y), \tag{2}$$

of degree 2, simply called a quadratic system in what follows. We denote by $\mathcal{X}:\mathbb{R}^2\to\mathbb{R}^2$ the vector field associated with the differential system (2), that is

$$\mathcal{X}(x,y)=(y,(1-\beta_1 y)(\gamma-\alpha x+\beta y)). \tag{3}$$

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