LIMIT CYCLES VIA HIGHER ORDER PERTURBATIONS FOR SOME PIECEWISE DIFFERENTIAL SYSTEMS

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ABSTRACT. A classical perturbation problem is the polynomial perturbation of the harmonic oscillator, $(x', y') = (-y + \varepsilon f(x, y, \varepsilon), x + \varepsilon g(x, y, \varepsilon))$. In this paper we study the limit cycles that bifurcate from the period annulus via piecewise polynomial perturbations in two zones separated by a straight line. We prove that, for polynomial perturbations of degree n, no more than $Nn - 1$ limit cycles appear up to a study of order N. We also show that this upper bound is reached for orders one and two. Moreover, we study this problem in some classes of piecewise Liénard differential systems. When we restrict the analysis to some special class this upper bound never is attained and we show which is this upper bound for higher order perturbation in ε. The Poincaré–Pontryagin–Melnikov theory is the main technique used to prove all the results.

1. INTRODUCTION

In last decades, piecewise differential systems have been useful for modeling real processes and different modern devices. For simplicity also the linear piecewise differential systems provides adequate models with very accurate results close to the observed data. See for more details in [7, 9]. Although in recent years these systems have attracted a good deal of attention, the first stages of modeling with piecewise systems started with Andronov and coworkers, see [2].

In this paper, we study the number of isolated periodic orbits, the so called *limit cycles*, of a polynomial piecewise perturbation of degree n of a linear center when the separation curve is a straight line Σ passing through the center. That is, we consider systems written as

$$(x', y') = \Big(-\frac{\partial H}{\partial y} + \sum_{i=1}^{N} \varepsilon^i f_i^{\pm}(x, y), \frac{\partial H}{\partial x} + \sum_{i=1}^{N} \varepsilon^i g_i^{\pm}(x, y) \Big), \tag{1}$$

such that the unperturbed system has a center at (x_c, y_c) with H a quadratic polynomial and the perturbations $f_i^{\pm}$ and $g_i^{\pm}$ are polynomials of degree n defined in each side of Σ. Via an affine change of coordinates, if necessary, it is not restrictive to assume that we are studying the piecewise polynomial perturbation of the harmonic oscillator. Consequently, we consider the above system in the form

$$(x', y') = \Big(-y + \sum_{i=1}^{N} \varepsilon^i f_i^{\pm}(x, y), x + \sum_{i=1}^{N} \varepsilon^i g_i^{\pm}(x, y) \Big), \tag{2}$$

with $f_i^{\pm}$ and $g_i^{\pm}$ polynomials of degree n defined in $\Sigma_0^{\pm} = \{(x, y) \in \mathbb{R}^2 : \pm y \geq 0\}$. This problem, for non piecewise perturbations, was studied by Iliev in [16], proving that the number of limit cycles is bounded by $[N(n-1)/2]$. In this paper, we prove by induction on the degree n that, in the piecewise case, the upper bound is approximately doubled. In the full paper $[\cdot]$ denotes the integer part function.

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