

No periodic orbits for the type A Bianchi's systems

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It is known that the 6 models of Bianchi class A have no periodic solutions. In this article we provide a new, direct, unified and easier proof of this result.

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1. Introduction

This paper deals with the Bianchi's cosmological models. These models require a three dimensional Lie algebra and Bianchi [1,2] was the first to solve the problem of classifying three dimensional Lie algebras. There are nine types of models according with the dimension n of the algebra.

- a) $n = 0$ Type *I*;
- b) $n = 1$ Type *II, III*;
- c) $n = 2$ Type *IV, V, VI, VII*;
- d) $n = 3$ Type *VIII, IX*.

Let $\{X_1, X_2, X_3\}$ be an appropriate basis of the three dimensional Lie Algebra. The classification depends on a scalar $a \in \mathbb{R}$ and a vector (n_1, n_2, n_3) with $n_i \in \{+1, -1, 0\}$ such that

$$[X_1, X_2] = n_3 X_3, \quad [X_2, X_3] = n_1 X_1 - a X_2, \quad [X_3, X_1] = n_2 X_2 + a X_1,$$

where $[\cdot, \cdot]$ is the Lie bracket. In particular, for $a = 0$ we obtain models of type *A* and for $a \neq 0$ we obtain models of type *B*. For more details see Bogoyavlensky [3].

According with [3] all cases of type *A* are Hamiltonian systems in the phase space p_i, q_i for $i = 1, 2, 3$ with the Hamiltonian function

$$H = \frac{1}{(q_1 q_2 q_3)^{\frac{1-k}{2}}} \left(T + \frac{1}{4} V_G \right),$$