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Qualitative Theory of Dynamical Systems

Phase Portraits of Reversible Linear Differential Systems with Cubic Homogeneous Polynomial Nonlinearities Having a Non-degenerate Center at the Origin

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Abstract. In this paper we classify the global phase portraits of all reversible linear differential systems with cubic homogeneous polynomial nonlinearities defined in the plane and having a non degenerate center at the origin. The reversibility is given by a linear involution having a fixed set of dimension 1.

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1. Introduction and setting the problem

We consider polynomial vector fields $X : \mathbb{R}^2 \to \mathbb{R}^2$ given by $X(x) = L(x) + g(x)$, where L is its linear part and g is a homogeneous polynomial vector field of degree three satisfying the following two hypotheses:

(H1) The vector field X is reversible with respect to a linear involution $\varphi : \mathbb{R}^2 \to \mathbb{R}^2$; (i.e., $\varphi \circ X = -X \circ \varphi$) such that the dimension of the fixed point set of φ is one.

(H2) The eigenvalues of L are $\pm\alpha i$ with $\alpha \neq 0$.

In this paper we classify the global phase portraits of all these vector fields X.

Cubic polynomial vector fields having a center have been investigated intensively, and many papers have been published about these systems, see for instance [3, 6, 12] and [13].

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