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Mathematics and Computers in Simulation 82 (2011) 533-539

www.elsevier.com/locate/matcom

Original article

## On the limit cycles of a class of piecewise linear differential systems in $\mathbb{R}^4$ with two zones

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> Received 15 June 2011; received in revised form 11 August 2011; accepted 30 August 2011 Available online 8 September 2011

## Abstract

We study the bifurcation of limit cycles from the periodic orbits of a four-dimensional center in a class of piecewise linear differential systems with two zones. Our main result shows that three is an upper bound for the number of limit cycles that bifurcate from a center, up to first order expansion of the displacement function. Moreover, this upper bound is reached. The main technique used is the averaging method.

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MSC: 58F14; 58F21; 58F30

Keywords: Limit cycles; Averaging theory; Piecewise linear systems with two zones

## 1. Introduction and statement of the main result

In the qualitative theory of differential equations the study of their limit cycles became one of the main topics. For a given differential system a *limit cycle* is a periodic orbit isolated in the set of all periodic orbits of the system.

Many questions arise on the limit cycles of the planar differential equations. Two main lines of research for such equations are, first the 16th Hilbert problem, see for instance [6,7], and second the study of how many limit cycles emerge from the periodic orbits of a center when we perturb it inside a given class of differential equations, see, for example, the book [3] and the references there in. More precisely, the problem of considering the planar linear differential center

 $\dot{x} = -y, \qquad \dot{y} = x,$ 

and perturbing it

 $\dot{x} = -y + \varepsilon P(x, y), \qquad \dot{y} = x + \varepsilon Q(x, y),$ 

inside a given class of polynomial differential equations and studying the limit cycles bifurcating from the periodic orbits of the linear center has attracted the interest and the research of many mathematicians. Of course,  $\varepsilon$  is a small

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<sup>0378-4754/\$36.00</sup> @ 2011 IMACS. Published by Elsevier B.V. All rights reserved. doi:10.1016/j.matcom.2011.08.006