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ORIGINAL PAPER

Hopf and zero-Hopf bifurcations in the Hindmarsh–Rose system

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Abstract We prove the existence of the classical Hopf bifurcation and of the zero-Hopf bifurcation in the Hindmarsh–Rose system. For doing this, some adequate change in parameters must be done in order that the computations become easier.

Keywords Hindmarsh–Rose burster · Hopf bifurcation · Periodic solutions

Mathematics Subject Classification 34C23 · 34C25 · 37G10

1 Introduction

These last years there was a big interest in studying the three-dimensional Hindmarsh–Rose polynomial ordinary differential system [1]. It appears as a reduction in the conductance based on the Hodgkin–Huxley model for neural spiking (see for more details [2]). This differential system can be written as:

$$\begin{aligned}
\dot{x} &= y - x^3 + bx^2 + I - z,\\
\dot{y} &= 1 - 5x^2 - y,\\
\dot{z} &= \mu(s(x - x_0) - z),
\end{aligned} \tag{1}$$

where $b,\ I,\ \mu,\ s,\ x_0$ are parameters and the dot indicates derivative with respect to the time t.

The interest in system (1) basically comes from two main reasons. The first one is due to its simplicity since it is just a differential system in $\mathbb{R}^3$ with a polynomial nonlinearity containing only five parameters. And the second one is because it captures the three main dynamical behaviors presented by real neurons: quiescence, tonic spiking and bursting. We can find in the literature many papers that investigate the dynamics presented by system (1) (see, for instance, [3–13]).

The study of the periodic orbits of a differential equation is one of the main objectives of the qualitative theory of differential equations. In general, the periodic orbits are studied numerically because, usually, their analytical study is very difficult. Here, using the averaging theory, we shall study analytically the periodic orbits of the three-dimensional Hindmarsh–Rose polynomial ordinary differential Eq. (1) which bifurcate firstly from a classical Hopf bifurcation and secondly from a zero-Hopf bifurcation.

Among the mentioned papers on the differential system (1), none of them study the occurrence of a Hopf or a zero-Hopf bifurcation in this differentiable sys-

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