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Bifurcation of limit cycles from a centre in $\mathbb{R}^4$ in resonance 1:N

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For every positive integer $N \geq 2$ we consider the linear differential centre $\dot{x} = Ax$ in $\mathbb{R}^4$ with eigenvalues $\pm i$ and $\pm Ni$. We perturb this linear centre inside the class of all polynomial differential systems of the form linear plus a homogeneous nonlinearity of degree N, i.e. $\dot{x} = Ax + \varepsilon F(x)$ where every component of $F(x)$ is a linear polynomial plus a homogeneous polynomial of degree N. Then if the displacement function of order ε of the perturbed system is not identically zero, we study the maximal number of limit cycles that can bifurcate from the periodic orbits of the linear differential centre.

Keywords: periodic orbits; limit cycles; polynomial vector fields; perturbation; resonance 1:N

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1. Introduction and statement of the main results

In the qualitative theory of differential equations the study of their limit cycles became one of the main topics. For a given differential equation $\mathcal{E}$ a *limit cycle* is a periodic orbit of $\mathcal{E}$ isolated in the set of all periodic orbits of $\mathcal{E}$.

Many questions arise on the limit cycles of the planar differential equations. Two main lines of research for such equations are, first the 16th Hilbert problem, see for instance [1,2], and second the study of how many limit cycles emerge from the periodic orbits of a centre when we perturb it inside a given class of differential equations, see, for example, the book [3] and the references there in. More precisely, the problem of considering the planar linear differential centre

$$\dot{x} = -y, \quad \dot{y} = x$$

and perturbing it

$$\dot{x} = -y + \varepsilon P(x, y), \quad \dot{y} = x + \varepsilon Q(x, y),$$

inside a given class of polynomial differential equations and studying the limit cycles bifurcating from the periodic orbits of the linear centre has attracted the interest and the

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