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Center boundaries for planar piecewise-smooth differential equations with two zones

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ABSTRACT

This paper is concerned with 1-parameter families of periodic solutions of piecewise smooth planar vector fields, when they behave like a center of smooth vector fields. We are interested in finding a separation boundary for a given pair of smooth systems in such a way that the discontinuous system, formed by the pair of smooth systems, has a continuum of periodic orbits. In this case we call the separation boundary as a *center boundary*. We prove that given a pair of systems that share a hyperbolic focus singularity p_0, with the same orientation and opposite stability, and a ray Σ_0 with endpoint at the singularity p_0, we can find a smooth manifold Ω such that $\Sigma_0 \cup \{p_0\} \cup \Omega$ is a center boundary. The maximum number of such manifolds satisfying these conditions is five. Moreover, this upper bound is reached.

1. Introduction

One of the most challenging problems in the qualitative theory of planar ordinary differential equations is the second part of the classical 16th Hilbert problem: the determination of an upper bound for the number of limit cycles for the class of polynomial vector fields of degree n. This problem remains unsolved if $n \geq 2$. The case $n = 1$ has a trivial answer because we can not have limit cycles for linear systems. By the other hand, we can have limit cycles for planar piecewise linear differential systems. It means that this problem, in the context of piecewise smooth systems, has attracted much attention.

The study of piecewise linear differential systems goes back to Andronov and coworkers [1]. These systems are used to model many real processes and different modern devices, see for more details [2] and the references therein.

The simplest case of piecewise linear differential systems is the one in which we have two half-planes separated by a straight line W. If both linear vector fields coincide at each point $w \in W$ we say that it

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