DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS
Volume **33**, Number **9**, September **2013**

doi:10.3934/dcds.2013.33.3915

pp. **3915–3936**

PIECEWISE LINEAR PERTURBATIONS OF A LINEAR CENTER

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(Communicated by Hinke Osinga)

Abstract. This paper is mainly devoted to the study of the limit cycles that can bifurcate from a linear center using a piecewise linear perturbation in two zones. We consider the case when the two zones are separated by a straight line Σ and the singular point of the unperturbed system is in Σ. It is proved that the maximum number of limit cycles that can appear up to a seventh order perturbation is three. Moreover this upper bound is reached. This result confirms that these systems have more limit cycles than it was expected. Finally, center and isochronicity problems are also studied in systems which include a first order perturbation. For the latter systems it is also proved that, when the period function, defined in the period annulus of the center, is not monotone, then it has at most one critical period. Moreover this upper bound is also reached.

1. **Introduction and main results.** The study of piecewise linear differential systems goes back to Andronov and coworkers [1], and in recent years these systems have attracted a good deal of attention. Piecewise linear differential systems are used to model many real processes and different modern devices, see for more details [3] and the references therein.

The case of continuous piecewise linear differential systems, when they have only two half-planes separated by a straight line is the simplest possible configuration of piecewise linear differential systems. In 1990, Lum and Chua conjectured that a continuous piecewise linear vector field in the plane with two zones has at most one limit cycle, see [25]. In 1998 this conjecture was proved by Freire, Ponce, Rodrigo and Torres in [12].

Limit cycles of discontinuous piecewise linear differential systems defined on two half-planes separated by a straight line have been studied recently in [5, 17, 18, 19, 22, 23, 24], among other papers. In [18], Han and Zhang provide discontinuous systems with two limit cycles, and they conjecture that the maximum number of

2010 *Mathematics Subject Classification.* Primary: 37G15, 34C05, 34C99.

Key words and phrases. Non-smooth differential system, limit cycle, piecewise linear differential system, centers, isochronicity.