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A Class of Reversible Cubic Systems with an Isochronous Center

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Abstract—We study cubic polynomial differential systems having an isochronous center and an inverse integrating factor formed by two different parallel invariant straight lines. Such systems are time-reversible. We find nine subclasses of such cubic systems, see Theorem 8. We also prove that time-reversible polynomial differential systems with a nondegenerate center have half of the isochronous constants equal to zero, see Theorem 3. We present two open problems.

Keywords—Differential equations, Cubic polynomial systems, Isochronous centers.

1. INTRODUCTION

It is known that the problem of isochronicity appears only for *nondegenerate centers*; i.e., centers whose linear part has nonzero imaginary eigenvalues. We consider here cubic polynomial differential systems (in what follows simply *cubic systems*) with a nondegenerate linear center at the origin. In an appropriate coordinate system and upon rescaling of the independent variable these systems take the form

$$\begin{aligned}\dot{x} &= -y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3,\\ \dot{y} &= x + b_{20}x^2 + b_{11}xy + b_{02}y^2 + b_{30}x^3 + b_{21}x^2y + b_{12}xy^2 + b_{03}y^3.\end{aligned} \tag{1}$$

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