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Darboux method and search of invariants for the Lotka–Volterra and complex quadratic systems

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The Darboux method introduces algebraic solutions quite useful to obtain invariant and first integrals of polynomial differential systems. Here we study the 2D Lotka–Volterra (LVS) and the complex quadratic system (QS) using straight lines for both and conics for the LVS. The conditions needed to obtain these invariants are given and a study of the phase space portrait is done. © *1999 American Institute of Physics.* [S0022-2488(99)02604-3]

I. INTRODUCTION

We consider the search of invariants for the two-dimensional differential system

$$\frac{dx}{dt}=P(x,y), \quad \frac{dy}{dt}=Q(x,y), \tag{1}$$

where P and Q are polynomials with coefficients in $\mathbf{F}$, where $\mathbf{F}$ is either the real field $\mathbf{R}$ or the complex field $\mathbf{C}$. We say that $m=\max\{\deg P,\deg Q\}$ is the *degree* of the polynomial differential system. This type of equations appear in the modelization of natural phenomena described in different branches of the science such as biology, chemistry, astrophysics, fluid mechanics, electronics, etc. Of particular interest are the systems such that $m=2$. The polynomial differential systems of degree 2 will be called *quadratic systems* (QS). One particularly well known quadratic system is the *Lotka-Volterra system* (LVS) which has been used to model the time evolutions of conflicting species in biology and of chemical reactions.[1,2] Among other applications, we find a QS in the equations of continuity describing the interactions of ions, electrons and neutral species in plasma physics (with the assumption of quasineutrality to eliminate either the ion or electron equation).[3] Moreover a reduced QS is obtained from a generalized Blasius equation for fluid flow around a wedge-shaped obstacle in boundary layer theory.[4] In the context of plasma physics, all the nonlinear terms represent binary interactions or model certain transport across the boundary of the system. There is a long history of research on finding sufficient conditions for which periodic solutions (center problem) exist for systems equivalent to the QS, and numerous results were obtained which we are not able to fully survey.[5] However, most of the previous works assumed that the origin is a linear center (i.e., having eigenvalues $\pm i$) which we do not assume here as starting point.

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