RENDICONTI DEL CIRCOLO MATEMATICO DI PALERMO
Serie II, Tomo LII (2003), pp. 389-418

LIOUVILLIAN FIRST INTEGRALS FOR THE PLANAR LOTKA–VOLTERRA SYSTEM

LAURENT CAIRÓ - HECTOR GIACOMINI - JAUME LLIBRE

We complete the classication of all Lotka–Volterra systems $\dot{x} = x(ax + by + c)$, $\dot{y} = y(Ax + By + C)$, having a Liouvillian first integral. In our classification we take into account the first integrals coming from the existence of exponential factors.

1. Introduction.

Nonlinear ordinary differential equations appear in many branches of applied mathematics and physics. For a 2–dimensional system the existence of a first integral determines completely its phase portrait. Since for such systems the notion of integrability is based on the existence of a first integral the following natural question arises: Given a system of ordinary differential equations depending on parameters, how to recognize the values of the parameters for which the system has a first integral? Of course, the more easiest planar integrable systems are the Hamiltonian ones. The planar integrable systems which are not Hamiltonian can be in general very difficult to detect. Many different methods have been used for studying the existence of first integrals for non–Hamiltonian systems based on: Noether symmetries [8], the Darboux theory of integrability [17], the Lie symmetries [38], the Painlevé analysis [3], the use of Lax pairs [27], the direct method [19, 24], the linear compatibility analysis method [44], the Carlemann embedding procedure [9, 2], the quasimonomial formalism [4], etc.

In 1878 Darboux [17] showed how can be constructed the first integrals of planar polynomial differential systems possessing sufficient invariant algebraic curves. In particular, he proved that if a planar polynomial differential system of degree m has at least $[m(m + 1)/2] + 1$ invariant algebraic curves, then