

Nonlinear Analysis 39 (2000) 351-363



www.elsevier.nl/locate/na

## Phase portraits of planar semi-homogeneous vector fields (II)

Laurent Cairó<sup>a</sup>, Jaume Llibre<sup>b,\*,1</sup>

<sup>a</sup> MAPMO-UMR 6628, Département de Mathématiques, Université d'Orleans, BP 6759, 45067 Orléans, Cedex 2, France

<sup>b</sup> Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 – Bellaterra, Barcelona, Spain

Received 12 March 1997; accepted 24 October 1997

Keywords: Phase portraits; Semi-homogeneous systems; Cubic systems

## 1. Introduction

Let *P* and *Q* be real homogeneous polynomials in the variables *x* and *y* of degrees *m* and *n*, respectively. Then we say that  $X = (P, Q) : \mathbb{R}^2 \to \mathbb{R}^2$  is a semi-homogeneous polynomial vector field, in particular if m = n, *X* is a homogeneous polynomial vector field.

The case m = n has been studied by several authors. The homogeneous quadratic polynomial vector fields (m = n = 2) by Lyagina [18], Markus [19], Korol [12], Sibirskii and Vulpe [22], Newton [21], Date [10] and Vdovina [23]; the homogeneous cubic polynomial vector fields (n = m = 3) by Cima and Llibre [6]; the general case m = n by Argemí [3], Cima and Llibre [6], Collins [9], Llibre, Pérez del Río and Rodriguez [15, 16]. In these papers is described an algorithm for studying the phase portraits of homogeneous polynomial vector fields for all degree m = n, the classification of all phase portraits of homogeneous polynomial vector fields and the characterization of structurally stable homogeneous and semi-homogeneous polynomial vector fields. In a previous paper [5], we study the phase portraits of the semi-homogeneous vector fields with m = 1 and n = 2.

<sup>\*</sup> Corresponding author. E-mail: jllibre@mat.uab.es.

<sup>&</sup>lt;sup>1</sup> The second author is partially supported by a DGICYT grant number PB96-1153.