

Phase Portraits of Planar Semi-Homogeneous Vector Fields (III)

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Abstract We determine all the phase portraits of the semi-homogeneous polynomial vector fields of the form $(ax + by, Ax^4 + Bx^3y + Cx^2y^2 + Dxy^3 + Ey^4)$.

Keywords Semi-homogeneous vector fields · Phase portraits

Mathematics Subject Classification (2000) Primary 34C05 · 34A34 · 34C14

1 Introduction

Let P and Q be real homogeneous polynomials in the variables x and y of degrees m and n , respectively. Then we say that $X = (P, Q) : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a *semi-homogeneous polynomial vector field*. In particular if $m = n$, X is a *homogeneous polynomial vector field*.

The case $m = n$ has been studied by several authors. The homogeneous quadratic polynomial vector fields ($m = n = 2$) by Lyagina [17], Markus [18], Korol [14], Sibirskii and Vulpe [21], Newton [20], Date [10] and Vdovina [22]; the homogeneous

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