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Research paper

Convergence regions for the Chebyshev–Halley family

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ABSTRACT

In this paper we study the dynamical behavior of the Chebyshev–Halley methods on the family of degree n polynomials $z^n + c$. We prove that, despite increasing the degree, it is still possible to draw the parameter space by using the orbit of a single critical point. For the methods having $z = \infty$ as an attracting fixed point, we show how the basins of attraction of the roots become smaller as the value of n grows. We also demonstrate that, although the convergence order of the Chebyshev–Halley family is 3, there is a member of order 4 for each value of n.

In the case of quadratic polynomials, we bound the set of parameters which correspond to iterative methods with stable behaviour other than the basins of attraction of the roots.

1. Introduction

Numerical methods are the main tool used by scientists and engineers for finding solutions of equations that can not be solved analytically. This encourages mathematicians, not only to seek new numerical methods, but also to study and improve the methods already known.

For the case of nonlinear equations, it is usual to consider iterative methods with high order of convergence to approximate the solutions. However, the radii of convergence which ensure that the solution of the method is correct are small. Accordingly, to improve the numerical methods and to expand their radii of convergence is one of the challenges to which mathematicians are faced. An overview of the analysis of different iterative methods can be found in [1]. In this book, besides studies dealing with the design, convergence, efficiency and robustness of the methods, we can also find an introduction to dynamical analysis of iterative methods.

The study of iterative methods from the point of view of dynamical systems gives a new perspective that allows to carry out a study of the qualitative behavior of such methods, establishing conditions of stability. It also enables us to study families of methods in terms of parameters, analyzing which members of the family work better.

The best known iterative method, under the dynamical point of view, is Newton's scheme (see [6]). In previous papers the dynamics of other iterative families are investigated: the Chebyshev–Halley family [12–14], the King's class [11], the c-family [9] and the (α, c)-family [8], which includes Chebyshev–Halley and c families. In these papers the dynamical behaviour of the iterative methods on quadratic polynomials is studied and regions with no convergence to the roots are found. Different authors have studied other point-to-point iterative methods for solving nonlinear equations (see [2–4,15,17],... for example).

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