Applied Mathematics and Computation 290 (2016) 326–335

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Newton method for symmetric quartic polynomial

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ARTICLE INFO

Keywords:
Newton's method
Holomorphic dynamics
Julia and Fatou sets

ABSTRACT

We investigate the parameter plane of the Newton's method applied to the family of quartic polynomials $p_{a,b}(z)=z^4+az^3+bz^2+az+1$, where a and b are real parameters. We divide the parameter plane $(a,b)\in\mathbb{R}^2$ into twelve open and connected *regions* where p, p' and p'' have simple roots. In each of these regions we focus on the study of the Newton's operator acting on the Riemann sphere.

1. Introduction

Newton's method is the universal root finding algorithm in all scientific areas of knowledge. It is also the seed of what we know as holomorphic dynamics and it goes back to Ernest Schröder and Artur Caley who investigated the global dynamics of Newton's method applied to low degree polynomials as a rational map defined on the Riemann sphere. This global study is not only theoretical but it also has important implications at computational level (see for instance [6]).

Given a rational map $f:\hat{\mathbb{C}}\to\hat{\mathbb{C}}$, where $\hat{\mathbb{C}}$ denotes the Riemann sphere, we consider the dynamical system given by the iterates of f. The Riemann sphere splits into two totally $f-$invariant subsets: the *Fatou set* $\mathcal{F}(f)$, which is defined to be the set of points $z\in\hat{\mathbb{C}}$ where the family $\{f^n,\ n\geq 1\}$ is normal in some neighborhood of z, and its complement, the *Julia set* $\mathcal{J}(f)=\hat{\mathbb{C}}\setminus\mathcal{F}(f)$. The Fatou set is open and therefore $\mathcal{J}(f)$ is closed. Moreover, if the degree of the rational map f is greater than or equal to 2, then the Julia set $\mathcal{J}(f)$ is not empty and it is the closure of the set of repelling periodic points of f.

The connected components of $\mathcal{F}(f)$, called *Fatou components*, are mapped under f among themselves. If follows from the Classification Theorem ([9], Theorem 13.1) that any periodic Fatou component of a rational map is either the basin of attraction of an attracting or parabolic cycle or a simply connected rotation domain (a Siegel disk) or a doubly connected component rotation domain (a Herman ring). Moreover, the basin of attraction of an attracting or parabolic cycle contains, at least, one critical point i.e. a point $z\in\hat{\mathbb{C}}$ such that $f'(z)=0$. For a background on the dynamics of rational maps we refer to [1,5,9].

Given p a polynomial of degree $d\geq 2$ we define the Newton's map as

$$N_p(z):=z-\frac{p(z)}{p'(z)}.$$

Clearly, roots of p correspond to attracting fixed points of N_p. It is well-known (see [11]) that $\mathcal{J}(N_p)$ is connected (see also [2,3]) and consequently, all Fatou components are simply connected. Although, as we claimed, Newton's method is *the*

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http://dx.doi.org/10.1016/j.amc.2016.06.021