J. Math. Anal. Appl. 462 (2018) 36–56

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Rational maps with Fatou components of arbitrarily large connectivity

Jordi Canela [1]

Institut de Mathématiques de Toulouse, Université Paul Sabatier, 118, route de Narbonne, F-31062 Toulouse, France

A R T I C L E I N F O

Article history:
Received 25 June 2017
Available online 2 February 2018
Submitted by P. Koskela

Keywords:
Holomorphic dynamics
Blaschke products
McMullen-like Julia sets
Singular perturbations
Connectivity of Fatou components

A B S T R A C T

We study the family of singular perturbations of Blaschke products $B_{a,\lambda}(z) = z^3 \frac{z-a}{1-\overline{a}z} + \frac{\lambda}{z^2}$. We analyse how the connectivity of the Fatou components varies as we move continuously the parameter λ. We prove that all possible escaping configurations of the critical point $c_-(a,\lambda)$ take place within the parameter space. In particular, we prove that there are maps $B_{a,\lambda}$ which have Fatou components of arbitrarily large finite connectivity within their dynamical planes.

1. Introduction

Given a rational map $f : \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$, where $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ denotes the Riemann Sphere, we consider the discrete dynamical system provided by the iterates of f. This dynamical system splits $\widehat{\mathbb{C}}$ into two totally invariant sets, the Fatou set $\mathcal{F}(f)$, which is defined as the set of points $z \in \widehat{\mathbb{C}}$ such that the family $\{f^n, n \in \mathbb{C}\}$ is normal in some neighbourhood of z, and its complement, the Julia set $\mathcal{J}(f)$. The dynamics of the points $z \in \mathcal{F}(f)$ is stable in the sense of normality whereas the dynamics of the points $z \in \mathcal{J}(f)$ presents a chaotic behaviour. The Fatou set $\mathcal{F}(f)$ is open and, hence, $\mathcal{J}(f)$ is closed. The connected components of $\mathcal{F}(f)$ are called Fatou components and are mapped under f among themselves. A Fatou component U is called periodic if there exists $q \in \mathbb{N}$ with $f^q(U) = U$, and preperiodic if there exists $q \in \mathbb{N}$ such that $f^q(U)$ is periodic. All Fatou components of rational maps are either periodic or preperiodic (see [21]). Moreover, any cycle of periodic Fatou components of a rational map has at least a critical point, i.e. a point $z \in \widehat{\mathbb{C}}$ such that $f'(z) = 0$, related to it. For a more detailed introduction to the dynamics of rational maps we refer to [4] and [14].

E-mail address: Jordi.Canela_Sanchez@math.univ-toulouse.fr.

[1] The author was supported by the ANR project LAMBDA, ANR-13-BS01-0002.

https://doi.org/10.1016/j.jmaa.2018.01.061
0022-247X/