Tongues in degree 4 Blaschke products

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Abstract

The goal of this paper is to investigate the family of Blasche products $B_a(z) = z^3 \frac{z-a}{1-\bar{a}z}$, which is a rational family of perturbations of the doubling map. We focus on the tongue-like sets which appear in its parameter plane. We first study their basic topological properties and afterwards we investigate how bifurcations take place in a neighborhood of their tips. Finally we see how the fixed tongue extends beyond its natural domain of definition.

Keywords: holomorphic dynamics, Blaschke products, circle maps, tongues
Mathematics Subject Classification numbers: 37F45, 37F10, 37F50, 37E10

(Some figures may appear in colour only in the online journal)

1. Introduction

Given a rational map $f : \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$, where $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ denotes the Riemann sphere, we consider the dynamical system given by the iterates of f. The Riemann sphere splits into two totally f-invariant subsets: the *Fatou set* $\mathcal{F}(f)$, which is defined to be the set of points $z \in \widehat{\mathbb{C}}$ where the family $\{f^n, n \in \mathbb{N}\}$ is normal in some neighborhood of z, and its complement, the *Julia set* $\mathcal{J}(f)$. The dynamics of the points in $\mathcal{F}(f)$ are stable in the sense of normality or equicontinuity, whereas the dynamics in $\mathcal{J}(f)$ present chaotic behavior. The Fatou set is open and its connected components, called *Fatou components*, are mapped under f among themselves.