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THE SET OF PERIODS FOR A CLASS OF SKEW–PRODUCTS

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Abstract. In this paper we give a characterization for the set of periods for a class of skew–products that we can see as deterministic systems driven by some stochastic process. This class coincides with a set of skew product maps from $\Sigma_N \times \mathbb{S}^1$ into itself, where Σ_N is the space of the bi–infinite sequences on N symbols and $\mathbb{S}^1$ is the unit circle.

1. **Introduction.** In 1970's Afraimovich and Shilnikov [1] described the semi–hyperbolic invariant set generated by a bifurcation of several homoclinic surfaces of a saddle–node cycle. The invariant set in the last bifurcation is homeomorphic to the product space $\Sigma_N \times \mathbb{S}^1$, where $\Sigma_N = \{0, 1, \ldots, N-1\}^{\mathbb{Z}}$ is the space of all bi–infinite sequences

$$\underline{a} = (\ldots, a_{-n}, \ldots, a_{-1} \cdot a_0, a_1, \ldots, a_n, \ldots) \tag{1.1}$$

of symbols $0, 1, \ldots, N-1$ (we note that in this paper we shall use the same notation as in [9]). The dynamics on the invariant set above, after some rescaling, give rise to a *skew product* as follows. Let $\underline{a} = (\ldots, a_{-1}.a_0, a_1 \ldots) \in \Sigma_N$ then $\sigma : \Sigma_N \to \Sigma_N$, *the shift map*, is given by

$$\sigma(\underline{a}) = (\ldots, a_{-1}, a_0 \cdot a_1, a_2, \ldots). \tag{1.2}$$

Let $\mathrm{Hom}(\mathbb{S}^1)$ be the set of homeomorphisms on $\mathbb{S}^1$ and let $\mathbb{G} \subset \mathrm{Hom}(\mathbb{S}^1)$ be an abelian group. For $\mathbf{f} = (f_0, f_1, \ldots, f_{N-1}) \in \mathbb{G}^N$ define the map

$$\Phi_{\mathbf{f}} : \Sigma_N \times \mathbb{S}^1 \to \Sigma_N \times \mathbb{S}^1 \tag{1.3}$$

given by

$$\Phi_{\mathbf{f}}(\underline{a}, x) = (\sigma(\underline{a}), f_{a_0}(x)). \tag{1.4}$$

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