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Zero-Hopf bifurcations in 3-dimensional differential systems with no equilibria

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Abstract

Recently sixteen 3-dimensional differential systems exhibiting chaotic motion and having no equilibria have been studied, and it has been graphically observed that these systems have a period-doubling cascade of periodic orbits providing a route to chaos. Here using new results on the averaging theory we prove that these systems exhibit, for some values of their parameters different to the ones having chaotic motion, either a zero-Hopf or a Hopf bifurcation, and graphically we observed that the periodic orbit starting in those bifurcations is at the beginning of the mentioned period-doubling cascade.

Keywords: Averaging theory; Periodic solutions; Quadratic polynomial differential system; Zero-Hopf bifurcation

1. Introduction and statement of the main results

In general the equilibria of a chaotic nonlinear system play an important role in its dynamics. In fact, one of the most important methods for obtaining 3-dimensional chaotic systems is the Shilnikov's method [10], which using a homoclinic orbit from the intersection of the stable and unstable manifolds of a saddle-focus equilibrium point with specified eigenvalues, provides the existence of a horseshoe in the neighborhood of this orbit and, consequently the existence of chaotic motion.

However some particularly important natural phenomena are described by nonlinear systems having no equilibria. Such as, the Noose–Hover oscillator [8], the Wei system [14] and the Wang–Chen system [13]. These nonlinear systems present chaotic behavior that cannot be detected by Shilnikov's method.

The increasing interest in finding examples of simple chaotic flows without equilibria has been motivating many researchers in recent times, see for instance [5,7,12,14,15]. The theoretical and practical importance of these systems converted this subject in a new attractive research direction. Although there is still little knowledge about the characteristics of such systems.

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