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Periodic Orbits Bifurcating from a Nonisolated Zero–Hopf Equilibrium of Three-Dimensional Differential Systems Revisited

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In this paper, we study the periodic solutions bifurcating from a nonisolated zero–Hopf equilibrium in a polynomial differential system of degree two in $\mathbb{R}^3$. More specifically, we use recent results of averaging theory to improve the conditions for the existence of one or two periodic solutions bifurcating from such a zero–Hopf equilibrium. This new result is applied for studying the periodic solutions of differential systems in $\mathbb{R}^3$ having n-scroll chaotic attractors.

Keywords: Averaging theory; periodic solutions; polynomial differential systems; zero–Hopf bifurcation; zero–Hopf equilibrium.

1. Introduction and Statement of the Main Result

In this paper, we study the periodic orbits bifurcating from a nonisolated zero–Hopf equilibrium of a three-dimensional autonomous differential system, it means that the differential system has a continuum of equilibria containing a point with a zero eigenvalue and a pair of purely imaginary eigenvalues. The zero–Hopf bifurcation is an interesting subject in differential systems and has been studied in [Guckenheimer, 1981; Guckenheimer & Holmes, 2013; Scheurle & Marsden, 1984; Kuznetsov, 2013] and by many other authors. Usually the zero–Hopf bifurcation is a two-parameter unfolding of a three-dimensional autonomous differential system with an isolated zero–Hopf equilibrium. It is known that some complicated invariant sets can bifurcate from an isolated zero–Hopf equilibrium, for instance zero–Hopf bifurcation can imply local birth of "chaos", see for instance [Scheurle & Marsden, 1984].

Most papers study isolated zero–Hopf equilibrium. One of the few works on nonisolated zero–Hopf equilibrium was done in 2012 by Llibre and Xiao [2014]. The authors studied the periodic orbits bifurcating from the nonisolated zero–Hopf equilibrium located at the origin of the following family of polynomial differential systems of degree two

$$\begin{aligned}\frac{dU}{dt} &= \varepsilon\lambda U - \omega V + \sum_{i+j+k=2} a_{ijk}(\varepsilon)U^iV^jW^k,\\ \frac{dV}{dt} &= \omega U + \varepsilon\lambda V + \sum_{i+j+k=2} b_{ijk}(\varepsilon)U^iV^jW^k,\\ \frac{dW}{dt} &= \varepsilon\mu W + \sum_{i+j+k=2} c_{ijk}(\varepsilon)U^iV^jW^k,\end{aligned} \quad (1)$$