PERSISTENCE OF PERIODIC SOLUTIONS FOR HIGHER ORDER PERTURBED DIFFERENTIAL SYSTEMS VIA LYAPUNOV-SCHMIDT REDUCTION

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Abstract. In this work we first provide sufficient conditions to assure the persistence of some zeros of functions having the form

$$g(z,\varepsilon)=g_0(z)+\sum_{i=1}^{k}\varepsilon^i g_i(z)+\mathcal{O}(\varepsilon^{k+1}),$$

for $|\varepsilon|\neq 0$ sufficiently small. Here $g_i:\mathcal{D}\to\mathbb{R}^n$, for $i=0,1,\ldots,k$, are smooth functions being $\mathcal{D}\subset\mathbb{R}^n$ an open bounded set. Then we use this result to compute the bifurcation functions which controls the periodic solutions of the following T–periodic smooth differential system

$$x'=F_0(t,x)+\sum_{i=1}^{k}\varepsilon^i F_i(t,x)+\mathcal{O}(\varepsilon^{k+1}),\quad (t,z)\in\mathbb{S}^1\times\mathcal{D}.$$

It is assumed that the unperturbed differential system has a sub-manifold of periodic solutions $\mathcal{Z}$, $\dim(\mathcal{Z})\leq n$. We also study the case when the bifurcation functions have a continuum of zeros. Finally we provide the explicit expressions of the bifurcation functions up to order 5.

1. Introduction

This work contains two main results. The first one (see Theorem A) provides sufficient conditions to assure the persistence of some zeros of smooth functions $g:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^n$ having the form

$$g(z,\varepsilon)=g_0(z)+\sum_{i=1}^{k}\varepsilon^i g_i(z)+\mathcal{O}(\varepsilon^{k+1}). \tag{1}$$

The second one (see Theorem B) provides sufficient conditions to assure the existence of periodic solutions of the following differential system

$$x'=F(t,z,\varepsilon)=F_0(t,x)+\sum_{i=1}^{k}\varepsilon^i F_i(t,x)+\mathcal{O}(\varepsilon^{k+1}),\quad (t,z)\in\mathbb{S}^1\times\mathcal{D}. \tag{2}$$

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