INVARIANT ALGEBRAIC SURFACES AND HOPF-BIFURCATION OF A FINANCE MODEL

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Abstract. Recently there are several works studying the finance model

$$\dot{x} = z + x(y - a), \quad \dot{y} = 1 - by - x^2, \quad \dot{z} = -x - cz$$

where a, b and c are positive parameters. The first objective of this paper is to show that this model exhibits one small amplitude periodic solution emerging from a Hopf bifurcation at the equilibrium point $(0, 1/b, 0)$ and in the second one we show that this system does not have invariant algebraic surfaces for any value of the parameters.

1. Introduction and statement of the main results

We consider the following polynomial differential system in $\mathbb{R}^3$

$$\begin{aligned} \dot{x} &= z + x(y - a), \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz, \end{aligned} \tag{1}$$

where a, b and c are real positive parameters and the dot denotes derivative with respect to the time t. This model has been intensively investigated (see, for instance, [1, 2, 4, 5, 8, 10]. It describes the time variation of these state variables: the interest rate x, the investment demand y and the price index z. Here a is the saving amount, b is the cost per investment and c is the elasticity of demand of commercial market. Changes in x come from an excess of investment over savings and the structural adjustment from good prices. Changes in y are in proportion to the rate of investment and to an inversion with the cost of investment and interest rates. Finally, changes in z are controlled by inflation rates.

The first objective of the present paper is to study the Hopf bifurcation which exhibits the polynomial differential system (1). We recall that a Hopf bifurcation in $\mathbb{R}^3$ takes place in an equilibrium point with eigenvalues of the form $\pm\omega i$ and λ, with $\omega, \lambda \in \mathbb{R}$. The Hopf bifurcation theory is well understood (see [6]). Our analysis of the Hopf bifurcation will be directly

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