A Local Spectral Inversion of a Linearized TV Model for Denoising and Deblurring

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***Abstract*—In this paper, we propose a model for denoising and deblurring consisting of a system of linear partial differential equations with locally constant coefficients, obtained as a local linearization of the total variation models [9]. The keypoint of our model is to get the local inversion of the Laplacian operator, which will be done *via* the Fast Fourier Transform (FFT). Two local schemes will be developed: a pointwise and a piecewise one. We will analyze both, their advantages and their limitations.**

***Index Terms*—Deblurring, image denoising, TV restoration.**

I. Introduction

DENOISING and deblurring are two of the most usual processes in image analysis. There is a huge amount of literature about these subjects from very different points of view. Some of these aspects, related to the model we are going to propose may be seen in [1], [2], [8], [9], [13] among others. There are two main classes of deblurring and denoising methods: linear and nonlinear. The first ones do not make use of the features of the signal, and they usually sacrifice quality of the results in order to get faster algorithms. Another big classification of the methods is locality *versus* globality. Solving locally a signal preserves it from external influences: errors, spurious oscillations … . Though they are not related, nonlinear methods tend to be local, while techniques for linear methods are usually global.

Due to the fact that they make use of the properties of the signals, nonlinear methods have been developed even for traditional linear techniques in order to improve the resolution of the methods. For instance, adaptive *wavelet* truncation, [3], is a clear example on how linear techniques, such as wavelets, are used in nonlinear schemes. Linear models are usually faster to evaluate and they are also used to obtain initial guesses for other methods.

In this paper, we are going to analyze a model for deblurring and denoising. This model consists of a system of partial differential equations (PDEs), obtained as a local linearization of a variational problem. In order to solve this model, a keypoint is the inversion of the Laplacian which appears in the Euler–Lagrange equations. Difference methods to solve such equations have stability restrictions, in particular due to the parabolic terms. The variational formulations solve the problem with constraints in a normed space, usually in the $\mathcal{L}^2$-norm, which presents the difficulties inherent to orthogonality, as Gibbs phenomena, for instance. In [9], the authors present a formulation in the total variation (TV) norm, which preserves edges. This formulation has computational difficulties due to the fact that the Euler–Lagrange equations are ill-conditioned. A regularization of those equations is presented in [5]. Our proposal is to use spectral methods to solve numerically the problem, by means of the Fast Fourier Transform, FFT. We will study its strength and its limitations. Though Fourier transform is global, we shall adapt it to a local resolution of the model. Furthermore, the analysis of the algorithms will show us that locality is closely related to the quality of the processed signals or images, and we will describe such relationship.

The basic idea of our method is to approximate the nonlinear model by means of a system of linear equations with locally constant coefficients. That is, we will freeze the coefficients in a decomposition of the initial domain, with overlapping boundaries, if necessary. The model has two parameters: a global regularizing parameter, a perturbation needed to smooth the functional, which is related to the variance of the noise in the signal and the other coefficient approximating the anisotropic diffusion.

Two special cases should be considered. When the decomposition is the whole domain we have the linear equation with constant coefficients, isotropic, with a global solution easily analyzed through Fourier transform. In the other case, which we call *pointwise*, the support consists of one point only, and the subdomains are considered as overlapping boundaries. The pointwise method is best adapted to the nonlinear model because, for every point, we use the coefficients without averaging. While the global model works very well for pure deconvolution, the pointwise one seems to be better fitted for pure denoising.

In some sense, deconvolution and denoising work in opposite directions: while denoising implies a smoothing of the signal, deconvolution sharpens it. This is the main reason why there are different models for both processes. A pure denoising model can deal with very noisy functions, but, if the signal is convolved, a slightly noisy signal becomes very difficult to deconvolve. The choice of the size of the support of the method implies a better or worse resolution. The balance between deconvolution and denoising is important to get a good processing, and it helps to determine the size of the support.

This paper is organized as follows. In the next section, we will describe the model, with its main properties, and we will propose the algorithms to solve the model. We shall develop in Section III the local model, which, to avoid confusion with

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