SCIENCE CHINA
Mathematics

• ARTICLES •

April 2014 Vol. 57 No. 4: 775–794
doi: 10.1007/s11425-014-4772-8

Darboux integrability and algebraic limit cycles for a class of polynomial differential systems

CAO JinLong[1], LLIBRE Jaume[2] & ZHANG Xiang[1,3,*]

[1]*Department of Mathematics, Shanghai Jiao Tong University, Shanghai* 200240*, China;*
[2]*Departament de Matemàtiques, Universitat Autònoma de Barcelona, Barcelona,* 08193 *Bellaterra, Spain;*
[3]*Ministry of Education Key Lab of Scientific and Engineering Computing, Shanghai Jiao Tong University, Shanghai* 200240*, China*
Email: jinlong@sjtu.edu.cn, jllibre@mat.uab.cat, xzhang@sjtu.edu.cn

Received December 27, 2012; accepted March 18, 2013; published online January 20, 2014

Abstract This paper deals with the existence of Darboux first integrals for the planar polynomial differential systems $\dot{x} = \lambda x - y + P_{n+1}(x,y) + xF_{2n}(x,y)$, $\dot{y} = x + \lambda y + Q_{n+1}(x,y) + yF_{2n}(x,y)$, where $P_i(x,y)$, $Q_i(x,y)$ and $F_i(x,y)$ are homogeneous polynomials of degree i. Within this class, we identify some new Darboux integrable systems having either a focus or a center at the origin. For such Darboux integrable systems having degrees 5 and 9 we give the explicit expressions of their algebraic limit cycles. For the systems having degrees 3, 5, 7 and 9 and restricted to a certain subclass we present necessary and sufficient conditions for being Darboux integrable.

Keywords Darboux first integral, algebraic limit cycles, Abel differential equation

MSC(2010) 34A34, 34C07

Citation: Cao J L, Llibre J, Zhang X. Darboux integrability and algebraic limit cycles for a class of polynomial differential systems. Sci China Math, 2014, 57: 775–794, doi: 10.1007/s11425-014-4772-8

1 Introduction and statement of the main results

There exist three main open problems in the qualitative theory of real planar differential systems, the distinction between a center and a focus, the determination of the number of limit cycles and their distribution, and the determination of its integrability. The importance for searching first integrals of a given system was already noted by Poincaré [26] in his discussion on a method to obtain polynomial or rational first integrals. One of the classical tools in the classification of all trajectories of a dynamical system is to find first integrals. Giné and Llibre [13, 14] characterized a large classes of polynomial differential systems in terms of the existence of Darboux first integrals. Llibre et al. [20,21] and Zhang [32] studied the exact upper bound of algebraic limit cycles of polynomial differential systems with the help of Darboux theory of integrability. In this paper, we will extend the results of [13, 14] to a new class of polynomial differential systems on their Darboux integrability and algebraic limit cyles.

We study the following systems:

$$\begin{aligned} \frac{dx}{dt} &= \dot{x} = \lambda x - y + P_{n+1}(x,y) + xF_{2n}(x,y) := P(x,y), \\ \frac{dy}{dt} &= \dot{y} = x + \lambda y + Q_{n+1}(x,y) + yF_{2n}(x,y) := Q(x,y), \end{aligned} \tag{1.1}$$

*Corresponding author

