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LIMIT CYCLES OF DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS

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We study the bifurcation of limit cycles from the periodic orbits of a two-dimensional (resp. four-dimensional) linear center in $\mathbb{R}^n$ perturbed inside a class of discontinuous piecewise linear differential systems. Our main result shows that at most 1 (resp. 3) limit cycle can bifurcate up to first-order expansion of the displacement function with respect to the small parameter. This upper bound is reached. For proving these results, we use the averaging theory in a form where the differentiability of the system is not needed.

Keywords: Discontinuous piecewise linear differential systems; limit cycles; averaging theory.

1. Introduction

The analysis of discontinuous piecewise linear differential systems goes back mainly to Andronov and coworkers [Andronov *et al.*, 1966] and nowadays still continues to receive attention by many researchers. In particular, discontinuous piecewise linear differential systems appear in a natural way in control theory and in the study of electrical circuits, see for instance [Bernardo *et al.*, 2008] and the references quoted there. These systems can present complicated dynamical phenomena such as those exhibited by general nonlinear differential systems. One of the main ingredients in the qualitative description of the dynamical behavior of a differential system is the number and the distribution of its limit cycles.

The goal of this paper is to study the existence of limit cycles of the control system of the form

$$\dot{x} = A_0 x + \varepsilon F(x), \tag{1}$$

with $|\varepsilon| \neq 0$ as a sufficiently small real parameter, where A_0 is equal to

$$A_0^1 = \begin{pmatrix} 0 & -1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \quad \text{or}$$

$$A_0^2 = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix},$$

and $F : \mathbb{R}^n \to \mathbb{R}^n$ is given by $F(x) = Ax + \varphi_0(k^T x)b$, with $A \in \mathcal{M}_n(\mathbb{R})$, $k, b \in \mathbb{R}^n \backslash \{0\}$ and $\varphi_0 : \mathbb{R} \to \mathbb{R}$