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Reversible periodic orbits in a class of 3D continuous piecewise linear systems of differential equations☆

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ABSTRACT

The so-called noose bifurcation is an interesting structure of reversible periodic orbits that was numerically detected by Kent and Elgin in the well-known Michelson system. In this work we perform an analysis of the periodic behavior of a piecewise version of the Michelson system where this bifurcation also exists. This variant is a one-parameter three-dimensional piecewise linear continuous system with two zones separated by a plane and it is also a representative of a wide class of reversible divergence-free systems.

In the piecewise system, the noose bifurcation involves reversible periodic orbits that intersect the separation plane at two or four points. This work is focused on those reversible periodic orbits that intersect the separation plane twice (RP2-orbits). It is established that for every T between 2π and a critical point, there exists a unique value of the parameter for which the system has an RP2-orbit with period T. Moreover, this critical value, that separates periodic orbits with two or four points of intersection with the separation plane, corresponds to an RP2-orbit that crosses the separation plane tangentially.

It is also proved that in a parameter versus period bifurcation diagram, the curve of this family of periodic orbits has a unique maximum point, which corresponds to the saddle-node bifurcation of periodic orbits that appears in the noose bifurcation.

1. Introduction

After equilibria, periodic orbits are the simplest solutions of nonlinear dynamical systems. In spite of that, generically, the proof of their existence is not a trivial problem.

Of course, there are local methods that allow us to affirm that there exist periodic orbits in neighborhoods of certain singularities of equilibria (Hopf, Bogdanov–Takens, …; see [1]) or close to global bifurcations (see, for instance, [2]). In the particular case of piecewise linear systems there are also results in this regard (see [3–7]). However, it is more difficult to answer some global questions as, for example, to determine the size of those neighborhoods in the phase space or in a parameter space, to establish where the periodic orbits born/die or to prove the existence of the homoclinic/heteroclinic cycle that organizes the periodic behavior.

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