Existence of homoclinic connections in continuous piecewise linear systems

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Numerical methods are often used to put in evidence the existence of global connections in differential systems. The principal reason is that the corresponding analytical proofs are usually very complicated. In this work we give an analytical proof of the existence of a pair of homoclinic connections in a continuous piecewise linear system, which can be considered to be a version of the widely studied Michelson system. Although the computations developed in this proof are specific to the system, the techniques can be extended to other piecewise linear systems. © *2010 American Institute of Physics*. [doi:10.1063/1.3339819]

The occurrence of a homoclinic orbit to a saddle-focus equilibrium satisfying certain eigenvalue condition assures the appearance of complex dynamics (Shil'nikov, 1965). Unfortunately, the proof of the existence of such an orbit is generically a difficult task and numerical techniques are often used. Arneodo, Coullet, and Tresser, in 1982, realized that piecewise linear systems gave a good chance of proving the existence of those dynamical objects and that Shil'nikov's result could be extended to this class of systems [see Arneodo *et al.* (1982) and Tresser (1984)]. In fact, as it is well known nowadays, piecewise linear systems are able to reproduce most of the dynamical behavior exhibited by general nonlinear systems. Furthermore, they are also becoming an important tool in the understanding of a wide range of dynamical phenomena in several areas of physics, engineering, and sciences in general. In this work, we present alternative conditions to those established in Arneodo *et al.* (1982) for the existence of a homoclinic connection in piecewise linear systems. Moreover, we give a complete analytical proof of the existence of a symmetrical pair of such connections in a continuous piecewise linear system which can be considered to be a version of the widely studied Michelson system.

I. INTRODUCTION

Homoclinic connections are orbits that are biasymptotic, for $t \to \pm\infty$, to the same equilibrium point. The existence of a homoclinic connection to a saddle-focus equilibrium point usually forces a complex dynamical behavior in a neighborhood of such connection. For instance, the celebrated works of Shil'nikov (Shil'nikov, 1965; Shil'nikov, 1970) assure, under certain eigenvalue ratio condition, the existence of infinitely many periodic orbits of saddle type accumulating to the homoclinic cycle.

An exhaustive recent revision of homoclinic connections for autonomous vector fields has been carried out in Homburg and Sandstede (2009). That work deals with the dynamic behavior related to the existence of homoclinic and heteroclinic orbits, the bifurcations of global connections, and the main analytical and geometric techniques used in their study. Other good works about theoretical and numerical aspects related to global connections are the pair of books (Shil'nikov *et al.*, 1998; 2001) and the survey (Champneys and Kuznetsov, 1994) which is more focused on the detection and continuation of global connections.

A large list of references about homoclinic connections and their bifurcations can also be found in these four previously cited works. Nevertheless, we would like to add here a short list of references about different topics regarding homoclinic cycles. For instance, several analyses of periodic motions near homoclinic connections (both in phase and parameter space) appear in Belyakov (1974); (1981); (1984), Gaspard *et al.* (1984), and Glendinning and Sparrow (1984).

The works (Devaney, 1976; 1978; Champneys, 1998; 1999) are devoted to global connections in reversible and Hamiltonian systems. The particular case of the restricted three-body problem is considered in Gómez *et al.* (1988).

Homoclinic connections and their bifurcations have also been reported and studied in nonsmooth systems (Arneodo *et al.*, 1982; Tresser, 1984) and partial differential equations (Feroe, 1981; Coullet, Riera, and Tresser, 2004; and Coullet, Toniolo, and Tresser, 2004). In fact, there are applications in many fields of science where homoclinic orbits have a special relevance (Gaspard *et al.*, 1993).

The principal problem in the study of homoclinic orbits is that a rigorous proof of its existence is generally a difficult task. One of the approaches of this problem is based on

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