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Existence of a Reversible T-Point Heteroclinic Cycle in a Piecewise Linear Version of the Michelson System*

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Abstract. The proof of the existence of a global connection in differential systems is generally a difficult task. Some authors use numerical techniques to show this existence, even in the case of continuous piecewise linear systems. In this paper we give an analytical proof of the existence of a reversible T-point heteroclinic cycle in a continuous piecewise linear version of the widely studied Michelson system. The principal ideas of this proof can be extended to other piecewise linear systems.

Key words. piecewise linear systems, heteroclinic orbits, invariant manifolds

AMS subject classifications. 34C23, 34C37, 37G99

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1. Introduction.

The existence of global connections in a differential system usually forces a complex dynamical behavior in a neighborhood of such connections. For instance, under the presence of a homoclinic cycle to a saddle-focus equilibrium point satisfying an eigenvalue ratio condition, the celebrated works of Shil'nikov [26, 27] ensure the existence of infinitely many periodic orbits of saddle type accumulating to the homoclinic cycle.

Moreover, the existence of a global connection in a differential system implies the appearance of subsidiary connections for certain perturbations of the system. For example, an analysis of the bifurcation structure of homoclinic cycles and subsidiary connections can be found in [15].

Heteroclinic cycles are also organizing centers of a very complex dynamic [10, 11, 12, 14]. In particular, Dumortier, Ibañez, and Kokubu [11] conjecture the existence of an infinite set of bifurcation phenomena, called a *cocoon* bifurcation [20], accumulating at a reversible T-point, that is, a point of the parameter space where a special kind of heteroclinic cycle satisfying some nondegeneracy condition appears. Furthermore, they explain the occurrence of such bifurcation phenomena as a consequence of the presence of this global connection.

Unfortunately, for nonlinear differential systems it is not easy to guarantee the existence of a global connection. Even though this is possible, some other extra conditions, for instance,

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