ON THE PERIODIC SOLUTIONS OF THE 5–DIMENSIONAL LORENZ EQUATION MODELING COUPLED ROSBY WAVES AND GRAVITY WAVES

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Abstract. Lorenz studied the coupled Rosby waves and gravity waves using the differential system

$$\begin{aligned}\dot{U} &= -VW + b\,VZ,\\ \dot{V} &= UW - bUZ,\\ \dot{W} &= -UV,\\ \dot{X} &= -Z,\\ \dot{Z} &= b\,UV + X.\end{aligned}$$

This system has the two first integrals

$$H_1 = U^2 + V^2,$$

$$H_2 = V^2 + W^2 + X^2 + Z^2.$$

Our main result shows in each invariant set $\{H_1 = h_1 > 0\} \cap \{H_2 = h_2 > 0\}$ there are at least 4 (respectively 2) periodic solutions of the differential system with $b \neq 0$ and $h_2 > h_1$ (respectively $h_2 < h_1$).

1. Introduction

In [3] E.N. Lorenz studied slow manifolds in coupled Rosby waves and gravity waves. In order to model the behavior he used the following 5-dimensional differential system

$$\begin{aligned}\dot{U} &= -VW + b\,VZ,\\ \dot{V} &= UW - bUZ,\\ \dot{W} &= -UV,\\ \dot{X} &= -Z,\\ \dot{Z} &= b\,UV + X,\end{aligned} \tag{1}$$

where $b \in \mathbb{R}$ is a parameter. In [4] it was studied the integrability of this differential system and, mainly its analytic integrability. In the present paper we shall use the first integrals of system (1) and the theory of averaging of first order for studying the periodic solutions of

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