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Limit cycles of discontinuous piecewise polynomial vector fields

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ABSTRACT

When the first average function is non-zero we provide an upper bound for the maximum number of limit cycles bifurcating from the periodic solutions of the center $\dot{x} = -y((x^2 + y^2)/2)^m$ and $\dot{y} = x((x^2 + y^2)/2)^m$ with $m \geq 1$, when we perturb it inside a class of discontinuous piecewise polynomial vector fields of degree n with k pieces. The positive integers m, n and k are arbitrary. The main tool used for proving our results is the averaging theory for discontinuous piecewise vector fields.

1. Introduction and statement of the main result

One of the main problems inside the qualitative theory of real planar differential systems is the determination of their limit cycles. The notion of a *limit cycle* of a planar differential system was defined by Poincaré [27] as a periodic orbit isolated in the set of all periodic orbits of the differential system. Van der Pol [28], Liénard [20] and Andronov [1] at the end of 1920s proved that a periodic orbit of a self-sustained oscillation occurring in a vacuum tube circuit was a limit cycle in the sense defined by Poincaré. After these results on the existence, non-existence and other properties of the limit cycles, these were studied with interest by mathematicians and physicists, and more recently also by many scientists of different areas (see for instance the books [10,32]).

In the last part of the XIX century Poincaré [27] defined the notion of a *center* of a real planar differential system, i.e. of an isolated equilibrium point having a neighborhood such that all the orbits of this neighborhood are periodic with the unique exception of the equilibrium point. Later on one way to produce limit cycles is by perturbing the periodic orbits of a center [29].

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